RESOURCE CONSTRAINED PROJECT SCHEDULING USING FUZZYAHP WITH TOPSIS

Dr. Lakshmi Tulasi Chavali[1], Professor & H.O.D., Dept. of Mechanical Engineering, St. Ann's College of Engineering & Technology, Chirala, India.

C. P. Balaji[2], Assistant Professor, Dept., of Mech., Engineering, Sri Venkateswara college of Engineering, Tirupati, India.

Duddela Niharika[3], Assistant Professor, G Narayanmma Institute of Technology and Science, Hyderabad, India.

Ravi Kumar Boddu[4], Assistant Professor, Dept. of Mechanical Engineering, St. Ann's College of Engineering & Technology, Chirala, India.

ABSTRACT

Several approaches are followed in project scheduling under multiple resources. Typically, priorities for each activity are obtained using qualitative data. In this paper, both quantitative and qualitative data are considered in a fuzzy environment. A Fuzzy Analytical Hierarchy process is developed to obtain weightages for each activity that needs multiple resources. Fuzzy Analytical Hierarchy Process efficiently handles the fuzziness of data. This paper develops an evaluation model based on the Fuzzy AHP and the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS). The Fuzzy AHP is employed to analyze the structure of the project and to determine the weights for the constraints, and TOPSIS method is used to develop the weights for the resources consumed by the activities. A weighted sum of resources for each activity is obtained using these weights of the resources and then ranking the activities by considering the weighted sum of the activities. Scheduling the activities is carried out taking into consideration the rank of the activity as well as the precedence relationship and resource requirements and the final project schedule is obtained. The method is demonstrated through numerical illustration.

***Keywords*:** FuzzyAHP, TOPSIS, Weighted sum, Project schedule.

INTRODUCTION

The project schedule is a tool that communicates what work needs to be performed, which resources of the organization will perform the work and the timeframes in which that work needs to be performed. The project schedule should reflect all of the work associated with delivering the project on time. Without a full and complete schedule, the project manager will be unable to communicate the complete effort, in terms of cost and resources, necessary to deliver the project.

Analytical hierarchy process (AHP) method developed by Satty (1981) has been widely used for selecting the best alternative. So the application of Satty's AHP has some shortcomings as follows (1) The AHP method is mainly used in nearly crisp decision applications, (2) The AHP method creates and deals with a very unbalanced scale of judgment, (3) The AHP method does not take into account the uncertainty associated with the mapping of one's judgment to a number, (4) Ranking of the AHP method is rather imprecise, (5) The subjective judgment, selection and preference of decision makers have great influence on the AHP results. It is also renowned that human assessment on qualitative attributes is always subjective and thus imprecise. Therefore conventional AHP seems not enough to confine decision maker's requirements clearly. In order to model this sort of uncertainty in human preference, fuzzy sets could be included with the pairwise comparison as an extension of AHP. The fuzzy AHP approach allows a more precise description of the decision making process. In this paper a fuzzy AHP approach is proposed to make up the ambiguity and uncertainty offered in the magnitude attributed to judgment of the decision maker, because the pair wise comparison in the conventional AHP seems to insufficient and imprecise to confine the degree of importance of decision maker. So, fuzzy logic is introduced in the pair wise comparison of AHP. The basis of resource constrained Project Scheduling using AHP with TOPSIS was given by (CH. Lakshmi Tulasi (2017)).

Fuzzy Analytical Hierarchy Process

The fuzzy AHP is the fuzzy addition of AHP to proficiently handle the fuzziness of the data drawn in the decision making. It is easier to understand and it can be effectively handle both qualitative and quantitative data in the multi-attribute decision making problems. In this approach triangular fuzzy numbers are used for the preferences of one criterion over another and then the pair wise comparison is calculated (Satty (1981)). To contract with vagueness of human thought (Zadeh (1965)) first proposed the fuzzy set theory, which was oriented to the rationality of uncertainty due to ambiguity or vagueness. A major contribution of fuzzy set theory is its ability of representing vague data. The theory also allows mathematical operators and programming to pertain to the fuzzy domain. A fuzzy set is a class of objects with a range of grades of membership. Such a set is characterized by a membership (characteristic) function, which assigns to each object a grade of membership ranging between zero and one. A tilde "~" will be positioned above a symbol if the symbol represents a fuzzy set. Though the purpose of AHP is to capture the expert's knowledge, the conventional AHP still cannot reflect the human thinking style. Therefore, fuzzy AHP, a fuzzy extension of AHP, was developed to solve the hierarchical fuzzy problems. In the fuzzy-AHP procedure, the pairwise comparisons in the judgment matrix are fuzzy numbers that are modified by the designer's emphasis (Kahraman et al. (2003)).

FUZZY REPRESENTATION OF PAIRWISE COMPARISON

The AHP method is also known as an eigenvector method. It indicates that the eigenvector consequent to the largest eigenvalue of the pairwise comparisons matrix provides the relative priorities of the factors, and preserves ordinal preferences among the alternatives. This means that if an substitute is preferred to another. A vector of weights obtained from the pairwise comparisons matrix reflects the relative performance of the various factors. Cheng and Mon (1994) explained that in the fuzzy AHP triangular fuzzy numbers are utilized to progress the scaling scheme in the judgment matrices, and interval arithmetic is used to solve the fuzzy eigenvector.

The four-step-procedure of this approach is given as

Step1: Comparing the performance score: triangular fuzzy numbers ($\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$) are used to indicate the relative strength of each pair of elements in the same hierarchy.

Step2: Constructing the fuzzy comparison matrix: By using triangular fuzzy numbers, via pairwise comparison, the fuzzy judgment matrix $\tilde{A}(a_{ij})$ is constructed as given below;

$$\tilde{A} = \begin{pmatrix} 1 & \widetilde{a_{12}} & \dots & \dots & \widetilde{a_{1n}} \\ \widetilde{a_{21}} & 1 & \dots & \dots & \widetilde{a_{2n}} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \widetilde{a_{n1}} & \widetilde{a_{n2}} & \dots & \dots & 1 \end{pmatrix}$$

Where, $\widetilde{a_{ij}^{\alpha}} = 1$, if I is equal j, and $\widetilde{a_{ij}^{\alpha}} = \tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$ or $\tilde{1}^{-1}$, $\tilde{3}^{-1}$, $\tilde{5}^{-1}$, $\tilde{7}^{-1}$, $\tilde{9}^{-1}$, if i is not equal j.

Step 3: Solving fuzzy eigenvalue: A fuzzy eigenvalue, $\tilde{\lambda}$ is a fuzzy number solution to

$\tilde{A}\tilde{x} = \tilde{\lambda}$ (1)

Where is n×n fuzzy matrix containing fuzzy numbers $\widetilde{a_{ij}}$ and $\tilde{x}$ is a non-zero nx1, fuzzy vector containing fuzzy number $\widetilde{x_i}$. To perform fuzzy multiplications and additions by using the interval arithmetic and α – cut, the equation $\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x}$ is equivalent to

$$[a_{i1l}^{\alpha}x_{1l}^{\alpha}, a_{i1u}^{\alpha}x_{1u}^{\alpha}] \oplus \cdots \oplus [a_{inl}^{\alpha}x_{nl}^{\alpha}, a_{inu}^{\alpha}x_{nu}^{\alpha}] = [\lambda x_{il}^{\alpha}, \lambda x_{iu}^{\alpha}]$$

Where,

$$\tilde{A} = [\widetilde{a_{ij}}],\ \ \widetilde{x^t} = (\widetilde{x_1}, \ldots \ldots, \widetilde{x_n})$$

$$\widetilde{a_{ij}}^{\alpha} = [a_{ijl}^{\alpha}, a_{iju}^{u}],\ \widetilde{x_i}^{\alpha} = [x_{il}^{\alpha}, x_{iu}^{\alpha}],\ \tilde{\lambda}^{\alpha} = [\lambda_l^{\alpha}, \lambda_u^{\alpha}] \quad (2)$$

for 0 < α ≤ 1 and all i, j, where i= 1,2,….,n, j = 1, 2, …., n

α – cut is known to incorporate the experts or decision maker(s) confidence over his/her preference or the judgments. Degree of satisfaction for the judgment matrix $\tilde{A}$ is estimated by the index of optimism μ. The larger value of index μ indicates the higher degree of optimism. The index of optimism is a linear convex combination (Lee et al., 2003) defined as

$$\widetilde{a_{iju}^{\alpha}} = \mu a_{iju}^{\alpha} + (1-\mu)a_{ijl}^{\alpha}, \qquad \forall \mu \in [0, 1] \quad (3)$$

While the α is fixed, the following matrix can be obtained after setting the index of optimism, μ, in order to estimate the degree of satisfaction.

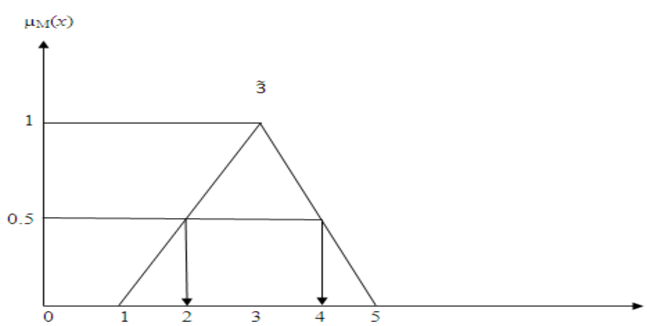

Figure 1: α – cut operation on triangular fuzzy number

$$\widetilde{A} = \begin{pmatrix} 1 & \widetilde{a_{12}} & \cdots & \cdots & \widetilde{a_{1n}^{\alpha}} \\ \widetilde{a_{21}^{\alpha}} & 1 & \cdots & \cdots & \widetilde{a_{2n}^{\alpha}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \widetilde{a_{n1}^{\alpha}} & \widetilde{a_{n2}^{\alpha}} & \cdots & \cdots & 1 \end{pmatrix}$$

The eigen vector is calculated by fixing the µ value and identifying the maximal eigenvalue. α – cut: It will yield an interval set of values from a fuzzy number. For example, α = 0.5 will yield a set $\alpha_{0.5}$ = (2,3,4). The operation is presented by using Table 5.12 (Figure 6.2).

Normalization of both the matrix of paired comparisons and calculation of priority weights (approx. attribute weights), and the matrices and priority weights for alternatives are also done before calculating λ_{max}. in order to control the result of the method, the consistency ratio for each of the matrices and overall inconsistency are expressed by the following equation CI, and the measure of inconsistency is called the CI,

$$CI = \frac{\lambda_{max} - n}{n-1} \tag{4}$$

The consistency ratio (CR) is used to estimate directly the consistency of pairwise comparisons. The CR is computed by dividing the CI by a value obtained from a table of Random Consistency Index (RI);

$$CR = \tag{5}$$

If the CR less than 0.10, the comparisons are acceptable, otherwise not. RI is the average index for randomly generated weights (Satty, 1981).

Step 4: The priority weight of each alternative can be obtained by multiplying the matrix of evaluation ratings by the vector of attribute weights and summing over all attributes. Expresses in conventional mathematical notation;

Weighted evaluation for alternative

$$k = \sum_{i=1}^{t} (attribute\ weight_i \times evaluation\ rating_{ik}) \tag{6}$$

For i= 1,2,…., t (t: total number of attributes)

After calculating the weight of each alternative, the overall consistency index is calculated to make sure that it is smaller than 0.10 for consistency on judgments.

TOPSIS METHODOLOGY

Step 1:

Establish a decision matrix for the ranking. The structure of the matrix can be expressed as follows

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_J \end{array} \begin{array}{c} \begin{array}{cccccc} F_1 & F_2 & \cdots & F_j & \cdots & F_n \end{array} \\ \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1j} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2j} & \cdots & f_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ f_{i1} & f_{i2} & \cdots & f_{ij} & \cdots & f_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ f_{J1} & f_{J2} & \cdots & f_{Jj} & \cdots & f_{Jn} \end{bmatrix} \end{array}$$

Where A_j denotes the alternatives j, j = 1,2,…,J; F_i represents i_{th} attribute or criterion, i= 1,2,…,n, related to i_{th} alternative; and f_{ij} is a crisp value indicating the performance rating of each alternative A_i with respect to each criterion F_j.

Step2: Calculate the normalized decision matrix R(=[r_{ij}]).The normalized value r_{ij} is calculated as :

$$r_{ij} = \frac{fij}{\sqrt{\sum_{j=1}^{J} f_{ij}^2}} \qquad j = 1,2,...,J;\ i = 1,2,...,n.$$

Step 3: Calculate the weighted normalized decision matrix by multiplying the normalized decision matrix by its associated weights. The weighted normalized value vij is calculated as:

$$V_{ij} = w_i \times r_{ij}, \quad j = 1,2,....,J; i = 1,2,...,n.$$

Where w_i represents the weight of the i_{th} attribute or criterion

Step4: Determine the positive-ideal and negative-ideal solutions

$$A^* = \left\{v_1^*, v_2^*, ..., v_i^*\right\}$$

$$= \left\{\left(\begin{matrix}\max v_{ij} \\ j\end{matrix} \middle| i \in I'\right), \left(\begin{matrix}\min v_{ij} \\ j\end{matrix} \middle| i \in I''\right)\right\},$$

$$A^- = \left\{v_1^-, v_2^-, ..., v_i^-\right\}$$

$$= \left\{\left(\begin{matrix}\min v_{ij} \\ j\end{matrix} \middle| i \in I'\right), \left(\begin{matrix}\max v_{ij} \\ j\end{matrix} \middle| i \in I''\right)\right\},$$

Where I' is associated with the benefit criteria, and I'' is associated with the cost criteria.

Step5: Calculate the separation measures, using the n-dimensional Euclidean distance. The separation of each alternative from the positive -ideal solution (D_j^*) is given as

$$D_j^* = \sqrt{\sum_{i=1}^{n} \left(v_{ij} - v_i^*\right)^2} \quad \text{j=1,2,….,J.}$$

Similarly, the separation of each alternative from the negative ideal solution (D_j^-) is as follows:

$$D_j^- = \sqrt{\sum_{i=1}^{n}\left(v_{ij} - v_i^-\right)^2} \text{ j=1, 2,…,J.}$$

Step 6: Calculate the relative closeness to the ideal solution and rank the performance order. The relative closeness of the alternative A_j can be expressed as

$$CC_j^* = \frac{D_j^-}{D_j^* + D_j^-} \text{, j=1,2,….,J.}$$

Where the CC_j^* index value lies between 0 and 1. The larger the index value means the better the performance of the alternatives.

Illustrative example

An illustrative example is taken to explain the scheduling of a project by using the methodologies Fuzzy AHP and TOPSIS.

Source: (Tarek Hegazy, (1999))

Activity	Duration (days)	Predecessors	Resource Requirements per day					
			R_1	R_2	R_3	R_4	R_5	R6
A	6	-	5	2	2	2	7	4
B	3	-	3	5	2	3	9	6
C	4	A	2	4	4	2	3	1
D	6	-	5	4	3	5	5	4
E	7	A,B	3	5	2	3	8	0
F	5	C	4	1	4	9	2	5
G	2	D	4	1	4	3	9	8
H	2	A,B	5	5	4	0	9	1
I	2	G,H	3	2	4	3	4	2
J	6	F	1	5	4	6	7	3
K	1	C,E	3	3	2	4	5	1
L	2	E,G,H	3	2	2	8	3	4
M	4	I,K	2	2	2	2	4	8
N	2	F,L	1	4	4	3	4	1
O	3	L	5	5	4	6	2	3
P	5	J,M,N	3	2	3	4	7	8

Q	8	O	4	5	4	2	3	4
R	2	D,O	5	3	3	3	7	8
S	6	P,R	2	4	6	2	3	4
T	2	Q	1	6	2	7	5	2
Daily Resource Limit			7	10	10	16	18	13

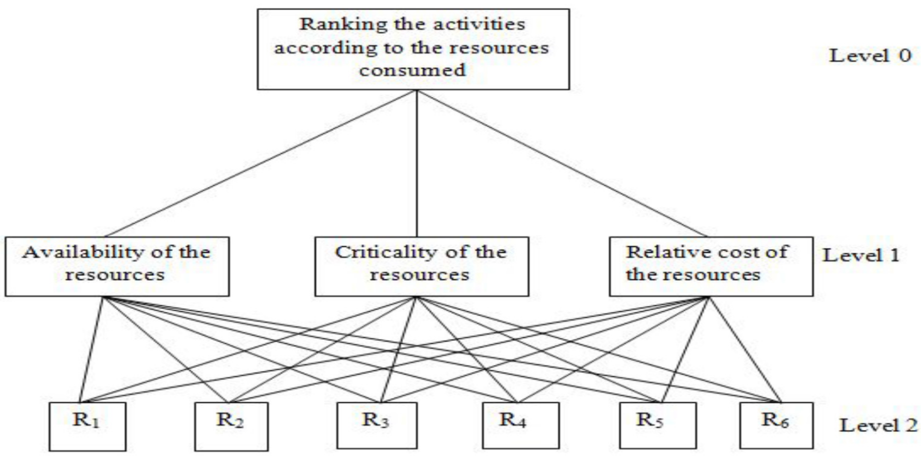

Figure 1 shows the hierarchy of the problem taking into consideration of the hierarchy of the problem applying the Fuzzy AHP and TOPSIS methodologies to obtain the weightages for the resources. Fuzzy AHP methodology is used to obtain the weightages for the criteria and TOPSIS is used to obtain the weightages for the resources with the use of the weightages of the criteria.

Table 2: calculating the fuzzy priority vector for the criteria

fuzzy comparison matrix					
Factors	**Availability**	**Criticality**	**Relative**		
Availability of	1	$\tilde{7}$	$\tilde{5}$		
Criticality of	$\tilde{7}^{-1}$	1	$\tilde{5}$		
Relative cost of	$\tilde{5}^{-1}$	$\tilde{5}^{-1}$	1		
α-cut fuzzy comparison matrix					
Availability of	1	[1, 2]	[4, 6]		
Criticality of	[1/2, 1]	1	[4, 6]		
Relative cost of	[1/6, 1/4]	[1/6, 1/4]	1		
Normalized fuzzy comparison matrix					
				Sum	Priority
Availability of	0.511	0.554	0.454	1.519	0.5068
Criticality of	0.383	0.369	0.454	1.206	0.4023
Relative cost of	0.106	0.077	0.091	0.274	0.091

Lambda max	3.0816				
Consistency	0.0408	n=3			
Consistency	0.0703				

Table 2 provides the weightages of the criteria obtained from the Fuzzy AHP methodology. After estimating the weightages for the criteria we are moving to estimate the weightages for the resources with TOPSIS methodology taking into consideration of weightages of the criteria. Calculating the weightages for the resources is shown in Table 3.

Table 3: Calculating the weightages of the resources using TOPSIS

	C_1	C_2	C_3	
	0.5068	0.4023	0.091	
R_1	3	3	3	
R_2	3	1	3	
R_3	5	5	3	
R_4	5	5	3	
R_5	7	5	7	
R_6	5	3	3	
Squared sum	142	94	94	
Normalized Decision matrix				
R_1	0.2518	0.3094	0.3094	
R_2	0.2518	0.1031	0.3094	
R_3	0.4196	0.5157	0.3094	
R_4	0.4196	0.5157	0.3094	
R_5	0.5874	0.5157	0.7720	
R_6	0.4196	0.3094	0.3094	
Weighted Normalized Matrix				
				Priority vector
R_1	0.1276	0.1245	0.0282	0.3249
R_2	0.1276	0.0415	0.0282	0.1364
R_3	0.2126	0.2075	0.0282	0.6910
R_4	0.2126	0.2075	0.0282	0.6910
R_5	0.2977	0.2075	0.0657	0.8636
R_6	0.2126	0.1245	0.0282	0.5119

Estimating the weighted sum for each activity using the weightages of the resources for each activity using the formula and ranking the activities according to the weighted sum of the activities.

Where i_j is the value of the resource required for the activity j; max_j and min_j are the maximum and minimum values of criterion j among all activities.

Table 4: weighted sum of the activities and ranking of the activities

Activity	Duration (days)	Predecessors	Resource Requirements per day						Weighted sum W_s	Rank
			R_1	R_2	R_3	R_4	R_5	R6		
			W_1	W_2	W_3	W_4	W_5	W_6		
			0.3249	0.1364	0.6910	0.6910	0.8636	0.5119		
A	6	-	5	2	2	2	7	4	1.3786	11
B	3	-	3	5	2	3	9	6	1.7493	4
C	4	A	2	4	4	2	3	1	0.8493	20
D	6	-	5	4	3	5	5	4	1.8361	2
E	7	A,B	3	5	2	3	8	0	1.242	13
F	5	C	4	1	4	9	2	5	1.6001	8
G	2	D	4	1	4	3	9	8	2.0738	1
H	2	A,B	5	5	4	0	9	1	1.707	6
I	2	G,H	3	2	4	3	4	2	1.1401	17
J	6	F	1	5	4	6	7	3	1.7241	5
K	1	C,E	3	3	2	4	5	1	1.2049	15
L	2	E,G,H	3	2	2	8	3	4	1.1832	16
M	4	I,K	2	2	2	2	4	8	1.0207	18
N	2	F,L	1	4	4	3	4	1	0.9682	19
O	3	L	5	5	4	6	2	3	1.4321	9
P	5	J,M,N	3	2	3	4	7	8	1.7983	3
Q	8	O	4	5	4	2	3	4	1.2311	14
R	2	D,O	5	3	3	3	7	8	1.6553	7
S	6	P,R	2	4	6	2	3	4	1.3869	10
T	2	Q	1	6	2	7	5	2	1.3095	12
Daily Resource Limit			7	10	10	16	18	13		

Table 4 gives the information about the weighted sum of the activities and ranking the activities as per their weighted sum. After ranking the activities arranging the activities as per their rank in the ascending order to ease the procedure of project scheduling based on the rank of the activities.

Table 5: Arranging the activities as per their rank in the ascending order

Rank	Activity	Predecessor	Duration (days)	R_1	R_2	R_3	R_4	R_5	R_6
1	G	D	2	4	1	4	3	9	8
2	D	-	6	5	4	3	5	5	4
3	P	J,M,N	5	3	2	3	4	7	8
4	B	-	3	3	5	2	3	9	6
5	J	F	6	1	5	4	6	7	3
6	H	A,B	2	5	5	4	0	9	1
7	R	D,O	2	5	3	3	3	7	8
8	F	C	5	4	1	4	9	2	5
9	O	L	3	5	5	4	6	2	3
10	S	P,R	6	2	4	6	2	3	4
11	A	-	6	5	2	2	2	7	4
12	T	Q	2	1	6	2	7	5	2
13	E	A,B	7	3	5	2	3	8	0
14	Q	O	8	4	5	4	2	3	4
15	K	C,E	1	3	3	2	4	5	1
16	L	E,G,H	2	3	2	2	8	3	4
17	I	G,H	2	3	2	4	3	4	2
18	M	I,K	4	2	2	2	2	4	8
19	N	F,L	2	1	4	4	3	4	1
20	C	A	4	2	4	4	2	3	1
Daily resource limit				7	10	10	16	18	13

Table 5 shows the arrangement of activities as per their rank in the ascending order. After arranging the activities as per their rank we proceed to scheduling of the project taking into consideration of rank of the activity, precedence relationship and the resources consumed by the activities. The project scheduling, critical path and the project duration are obtained from the Gantt chart (Figure 2).

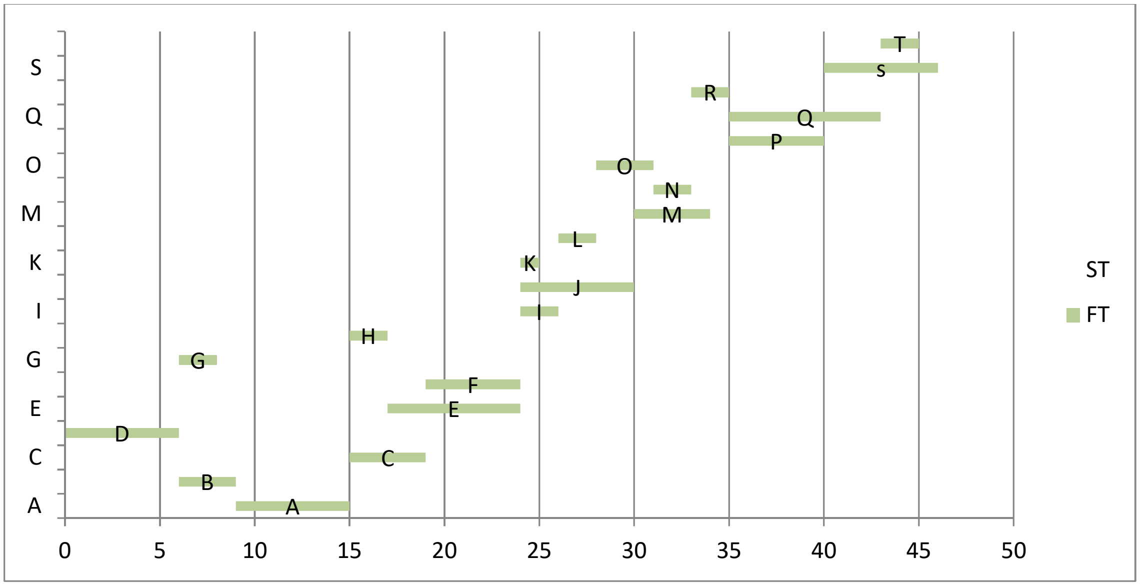

Figure 2: Gantt chart for Fuzzy AHP with TOPSIS

Critical path: D-B-A-C-F-I-L-O-N-R-P-S (OR) D-B-A-H-E-I-L-O-N-R-P-S

Project duration: 46 days`

CONCLUSION

The insightful ability and methods to compose sound decisions are implicated in the complex decision making situation in the project management. The project scheduling with the help of Fuzzy AHP with TOPSIS in developing the weightages for the activities are done in this paper and the corresponding project schedule is shown in this paper. This paper provides the source for applications of Fuzzy AHP with TOPSIS weightages in the project scheduling. The final project scheduling and the project duration are obtained from the Gantt chart.

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