$\tilde{\gamma}$-separation axioms, $(\gamma, \tilde{\gamma})$-normal and $(\gamma, \tilde{\gamma})$-regular spaces

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Abstract

In this paper, we create a new type of separation spaces $\tilde{\gamma}$-R_0 and $\tilde{\gamma}$-R_1 through $\tilde{\gamma}$-open sets and study some of their characterizations and relationships between $\tilde{\gamma}$-T_0, $\tilde{\gamma}$-T_1 and $\tilde{\gamma}$-T_2 spaces. Also, we introduce $(\gamma, \tilde{\gamma})$-normal and $(\gamma, \tilde{\gamma})$-regular spaces and study their important properties through operation mappings.

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1 Introduction

Kasahara[14] introduced the concept of operation topology, subsequently Jankovic[10] developed the operation-closed graphs. Ogata[20, 21] defined γ-operation on a topology and investigated the relationships between γ-closure and τ_γ-closure operators and obtained the concept of γ-T_i $(i = 0, \frac{1}{2}, 1, 2)$ spaces through γ-closed and γ-open sets. Umehara et.al.[36] and Ogata[22] discussed the bioperation concept in operation topology with some separation axioms. Levine[16] defined the notion of generalized closed set in topology and Maki et.al.[18] created a γg-closed set in operation topology. Dunham[9] introduced $T_{\frac{1}{2}}$ spaces and Levine[15] defined a semi-open set in topology. Sai Sundara Krishnan et.al.[23] modified the concept of semi-open as γ-semi-open in operation topology and studied the γ-semi-separation axioms. Saravanakumar et.al.[24, 29, 32] initiated a $\tilde{\gamma}$-open set (resp. γ^*-pre-open) set in operation topology and $\tilde{\mu}$-open set in general topology and studied $\tilde{\gamma}$-T_i (resp. γ^*-pre-T_i, $\tilde{\mu}$-T_i), $(i = 0, \frac{1}{2}, 1, 2)$ spaces using through the $\tilde{\gamma}$-open (resp. γ^*-pre-open, $\tilde{\mu}$-open) and $\tilde{\gamma}$-closed (resp. γ^*-pre-closed, $\tilde{\mu}$-closed) sets. Saravankumar et.al.[25, 27, 30, 32, 33, 34, 35] created various types of operation continuous mappings in operation topology as well as generalized continuous in general topology and discussed some important properties. In [1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 34, 36, 37], they introduced new type of separation axioms such as κ-T_i $(i = 0, \frac{1}{2}, 1, 2)$, κ-R_j $(j = 0, 1)$, κ-normal and κ-regular spaces (here "κ" stands for semi, pre, $\wedge_\theta$, μ, γ, γ^*-pre, $\tilde{\gamma}$ etc.) in the fields of general topology, generalized topology, operation topology and discussed some of their relationships and properties.

In this paper, we introduced new notion of operation separation axioms $\tilde{\gamma}$-R_0 and $\tilde{\gamma}$-R_1 spaces and obtained that every $\tilde{\gamma}$-R_1 space is $\tilde{\gamma}$-R_0, but the converse need not be true. Also, we investigated their relationships between $\tilde{\gamma}$-T_0, $\tilde{\gamma}$-T_1 and $\tilde{\gamma}$-T_2 spaces and studied some of their important properties. Moreover, we defined the concepts of $(\gamma, \tilde{\gamma})$-normal and $(\gamma, \tilde{\gamma})$-regular spaces and discussed the characterizations through $\tilde{\gamma}$-open and $\tilde{\gamma}$-closed sets. In addition, we proved that topological space X is $(\gamma, \tilde{\gamma})$-normal (resp. $(\gamma, \tilde{\gamma})$-regular) if $f : X \to Y$ is a (γ, β)-closed and $(\tilde{\gamma}, \tilde{\beta})$-continuous, injective mapping and Y is a $(\beta, \tilde{\beta})$-normal (resp. $(\beta, \tilde{\beta})$-regular) space.

2 Preliminaries

Throughout this paper, we consider the topological spaces (X, τ) and (Y, σ) by X and Y resp. An operation γ[20] on the topology τ is a mapping from τ into the power set $P(X)$ of X such that $V \subseteq V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of γ at V. Similarly, an operation β on the topology σ is a mapping from σ into the power set $P(Y)$ of Y such that $W \subseteq W^\beta$ for each $W \in \sigma$, where W^β denotes the value of β at W. A subset A of X is γ-open[20], if

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for each $x \in A$, there exist an open neighborhood U such that $x \in U$ and $U^{\gamma} \subseteq A$. Its complement is called γ-closed and τ_{γ}[20] denotes set of all γ-open sets in X. For a subset A of X, γ-interior[20] of A is $int_{\gamma}(A) = \{x \in A : x \in N \in \tau \text{ and } N^{\gamma} \subseteq A \text{ for some } N\}$; γ-closure[20] of A is $cl_{\gamma}(A) = \{x \in X : x \in U \in \tau \text{ and } U^{\gamma} \cap A \neq \emptyset \text{ for all } U\}$; τ_{γ}-$int(A)$[20] $= \cup\{G : G \subseteq A \text{ and } G \in \tau_{\gamma}\}$; τ_{γ}-$cl(A)$[20] $= \cap\{F : A \subseteq F \text{ and } X \setminus F \in \tau_{\gamma}\}$. A subset A of X is γg-closed[20] if $cl_{\gamma}(A) \subseteq U$ whenever $A \subseteq U$ and U is γ-open in X. For a subset A of X, $cl^{*}_{\gamma}(A)$[18] denotes the intersection of all γg-closed sets containing A, that is the smallest γg-closed set containing A; $int^{*}_{\gamma}(A)$[18] denotes the union of all γg-open sets contained in A, that is the largest γg-open set contained in A. If A is a subset of X and $x \in X$, then (i) $x \in cl^{*}_{\gamma}(A)$[18] if and only if $M \cap A \neq \emptyset$ for each γg-open set M containing x; (ii) $cl^{*}_{\gamma}(X \setminus A)$[18] $= X \setminus int^{*}_{\gamma}(A)$ and (iii) $cl^{*}_{\gamma}(cl^{*}_{\gamma}(A))$[18] $= cl^{*}_{\gamma}(A)$. A subset A of X is γ-semi-open[23] if $A \subseteq \tau_{\gamma}cl(\tau_{\gamma}int(A))$ and $\gamma SO(X)$[23] denotes the family of all γ-semi-open sets in X. A subset A of X is said to be a $\tilde{\gamma}$-open set[32], if there exists a set $U \in \tau_{\gamma}$ such that $U \subseteq A \subseteq cl^{*}_{\gamma}(U)$. Its complement is called $\tilde{\gamma}$-closed. The family of all $\tilde{\gamma}$-open sets is denoted by $\tilde{\gamma}O(X)$. For $A \subseteq X$, $\tilde{\gamma}$-interior of A[32] is $int_{\tilde{\gamma}}(A) = \cup\{U : U \in \tilde{\gamma}O(X) \text{ and } U \subseteq A\}$ and $\tilde{\gamma}$-closure of A[32] is $cl_{\tilde{\gamma}}(A) = \cap\{F : X - F \in \tilde{\gamma}O(X) \text{ and } A \subseteq F\}$. A subset A of X is said to be $\tilde{\gamma}g$-closed[32] if $cl_{\tilde{\gamma}}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tilde{\gamma}$-open in X. A subset A of X is said to be $\tilde{\gamma}g$-open[32] if $F \subseteq int_{\tilde{\gamma}}(A)$ whenever $F \subseteq A$ and F is $\tilde{\gamma}$-closed in X. The family of all $\tilde{\gamma}g$-open sets is denoted by $\tilde{\gamma}GO(X)$. A space X is said to be (i) $\tilde{\gamma}$-T_0[32] if for each pair of distinct points $x, y \in X$, there exists a $\tilde{\gamma}$-open set U such that $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$; (ii) $\tilde{\gamma}$-T_1[32] if for each pair of distinct points $x, y \in X$, there exists a $\tilde{\gamma}$-open sets U and V contain x and y respectively such that $y \notin U$ and $x \notin V$; (iii) $\tilde{\gamma}$-T_2[32] if for each pair of distinct points $x, y \in X$, there exists a $\tilde{\gamma}$-open sets U and V such that $x \in U$ and $y \in V$ and $U \cap V = \emptyset$. A mapping $f : X \to Y$ is said to be (γ, β)-open[20] (resp. (γ, β)-closed[20]) if for each γ-open set U (resp. γ-closed) of X, $f(U)$ is β-open (resp. β-closed) in Y. A mapping $f : X \to Y$ is said to be (γ, β)-continuous[20] (resp. $(\tilde{\gamma}, \tilde{\beta})$-continuous[32]) if for any β-open V (resp. $\tilde{\beta}$-open) of Y, $f^{-1}(V)$ is γ-open (resp. $\tilde{\gamma}$-open) in X.

Definition 2.1. For a subset A of X, $ker_{\tilde{\gamma}}(A) = \cap\{U : U \in \tilde{\gamma}O(X) \text{ and } A \subseteq U\}$ is called $\tilde{\gamma}$-kernel of A.

Definition 2.2. A subset A of a topological space X is said to be $g\tilde{\gamma}$-closed if $cl_{\tilde{\gamma}}(A) \subseteq U$ whenever $A \subseteq U$ and U is γ-open in X. A subset A of a topological space X is said to be $g\tilde{\gamma}$-open if $F \subseteq int_{\tilde{\gamma}}(A)$ whenever $F \subseteq A$ and F is γ-closed in X. The family of all $g\tilde{\gamma}$-open sets is denoted by $G\tilde{\gamma}O(X)$.

Definition 2.3. A mapping $f : X \to Y$ is said to be $(\tilde{\gamma}, \tilde{\beta})$-open, (resp. $(\tilde{\gamma}, \tilde{\beta})$-closed if for each $\tilde{\gamma}$-open set U (resp. $\tilde{\gamma}$-closed) of X, $f(U)$ is $\tilde{\beta}$-open (resp. $\tilde{\beta}$-closed) in Y.

Remark 2.1[32]. Let X be a topological space. Then for a point $x \in X$, $x \in cl_{\tilde{\gamma}}(A)$ if and only if $V \cap A \neq \emptyset$ for any $V \in \tilde{\gamma}O(X)$ such that $x \in V$.

Remark 2.2 Let X be a topological space. If A is a $\tilde{\gamma}$-open set in X, then A is $g\tilde{\gamma}$-open in X.

Lemma 2.1. The following properties hold for subsets A, B of a topological space X:
(i) $x \in ker_{\tilde{\gamma}}(A)$ if and only if $A \cap F \neq \emptyset$ for any $\tilde{\gamma}$-closed set F of X containing x;
(ii) $A \subseteq ker_{\tilde{\gamma}}(A)$ and $A = ker_{\tilde{\gamma}}(A)$ if A is $\tilde{\gamma}$-open in X;
(iii) if $A \subseteq B$, then $ker_{\tilde{\gamma}}(A) \subseteq ker_{\tilde{\gamma}}(B)$.

Proof. Follows from the Definition 2.1.

Lemma 2.2. For $A \subseteq X$, $ker_{\tilde{\gamma}}(A) = \{x \in X : cl_{\tilde{\gamma}}(\{x\}) \cap A \neq \emptyset\}$.

Proof. Let $x \in ker_{\tilde{\gamma}}(A)$. If $cl_{\tilde{\gamma}}(\{x\}) \cap A = \emptyset$, then $x \notin X - cl_{\tilde{\gamma}}(\{x\})$, which is a $\tilde{\gamma}$-open set containing A. Thus $x \notin ker_{\tilde{\gamma}}(A)$, a contradiction. Hence $cl_{\tilde{\gamma}}(\{x\}) \cap A \neq \emptyset$. Conversely, let $x \in X$ be such that $cl_{\tilde{\gamma}}(\{x\}) \cap A \neq \emptyset$. If possible, let $x \notin ker_{\tilde{\gamma}}(A)$. Then there exists $U \in \tilde{\gamma}O(X)$ such that $x \notin U$ and $A \subseteq U$. Let $y \in cl_{\tilde{\gamma}}(\{x\}) \cap A$. Then $y \in cl_{\tilde{\gamma}}(\{x\})$ and $y \in U$, which gives $x \in U$, a contradiction. Hence $x \in ker_{\tilde{\gamma}}(A)$.

3 $\tilde{\gamma}$-R_i spaces

Definition 3.1. A topological space X is said to be $\tilde{\gamma}$-R_0 if for each $\tilde{\gamma}$-open set U, $x \in U$ implies that $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$

Example 3.1. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ and define an operation $\gamma : \tau \to P(X)$ by

$$A^\gamma = \begin{cases} A \cup \{a\} & \text{if } A = \{c\} \\ cl(A) & \text{if } A \neq \{c\} \end{cases} \quad \text{for every } A \in \tau.$$

Then $\tilde{\gamma}O(X) = \{\emptyset, X, \{a\}, \{b, c\}\}$. Hence X is $\tilde{\gamma}$-R_0.

Theorem 3.1. Let X be a topological space and $x, y \in X$. Then $y \in ker_{\tilde{\gamma}}(\{x\})$ if and only if $x \in cl_{\tilde{\gamma}}(\{y\})$

Proof. Let $y \in ker_{\tilde{\gamma}}(\{x\})$. If $x \notin cl_{\tilde{\gamma}}(\{y\})$, then $x \notin \cap\{F : X - F \in \tilde{\gamma}O(X)$ and $\{y\} \subseteq F\}$ implies that $x \in X - F$ and $y \notin X - F$. Therefore $y \notin \cap\{X - F : X - F \in \tilde{\gamma}O(X)$ and $\{x\} \subseteq X - F\}$ and hence $y \notin ker_{\tilde{\gamma}}(\{x\})$, which is a contradiction. Thus $x \in cl_{\tilde{\gamma}}(\{y\})$. Conversely, let $x \in cl_{\tilde{\gamma}}(\{y\})$. If $y \notin ker_{\tilde{\gamma}}(\{x\})$, then $y \notin \cap\{U : U \in \tilde{\gamma}O(X)$ and $\{x\} \subseteq U\}$ implies that $y \in X - U$ and $x \notin X - U$. Therefore $x \notin \cap\{X - U : U \in \tilde{\gamma}O(X)$ and $\{y\} \subseteq X - U\}$ and hence $x \notin cl_{\tilde{\gamma}}(\{y\})$, a contradiction. Thus $y \in ker_{\tilde{\gamma}}(\{x\})$.

Theorem 3.2. In a topological space X, the following statements are equivalent:
(i) X is $\tilde{\gamma}$-R_0;
(ii) for each $\tilde{\gamma}$-closed set F and a point $x \notin F$, there exists a $G \in \tilde{\gamma}\text{O}(X)$ such that $x \notin G$ and $F \subseteq G$;
(iii) for each $\tilde{\gamma}$-closed set F and $x \notin F$, $cl_{\tilde{\gamma}}(\{x\}) \cap F = \emptyset$.

Proof. (i) $\Rightarrow$ (ii). Let F be $\tilde{\gamma}$-closed and $x \notin F$. Then $X - F$ is $\tilde{\gamma}$-open and $x \in X - F$. By (i) $cl_{\tilde{\gamma}}(\{x\}) \subseteq X - F$, $X - F$ is $\tilde{\gamma}$-open and $x \in X - F$. Let $G = X - cl_{\tilde{\gamma}}(\{x\})$ is $\tilde{\gamma}$-open. Since $x \in cl_{\tilde{\gamma}}(\{x\}) \Rightarrow x \notin X - cl_{\tilde{\gamma}}(\{x\}) \Rightarrow x \notin G$.

(ii) $\Rightarrow$ (iii). Let F be a $\tilde{\gamma}$-closed set and $x \notin F$. Then by (ii), there exists a $\tilde{\gamma}$-open set $G, x \notin G$ and $F \subseteq G$. Thus $x \in X - G \subseteq X - F$. Hence $X - G$ is $\tilde{\gamma}$-closed containing x and we have that $cl_{\tilde{\gamma}}(\{x\}) \cap (X - G) \neq \emptyset$ implies that $cl_{\tilde{\gamma}}(\{x\}) \subseteq X - G$. Therefore $G \cap cl_{\tilde{\gamma}}(\{x\}) = \emptyset$ and hence $F \cap cl_{\tilde{\gamma}}(\{x\}) = \emptyset$.

(iii) $\Rightarrow$ (i). Let F is $\tilde{\gamma}$-closed $x \notin F$, $cl_{\tilde{\gamma}}(\{x\}) \cap F = \emptyset$. Let U be $\tilde{\gamma}$-open and $x \in U$. Then $X - U$ is $\tilde{\gamma}$-closed and $x \notin X - U$. By (iii), $cl_{\tilde{\gamma}}(\{x\}) \cap (X - U) = \emptyset$ implies that $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$. Hence X is $\tilde{\gamma}$-R_0.

Theorem 3.3. A topological space X is $\tilde{\gamma}$-R_0 if and only if for each pair of $x, y \in X$ and $x \neq y$, $cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\}) = \emptyset$ (or) $\{x, y\} \subseteq cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\})$.

Proof. Let X be a $\tilde{\gamma}$-R_0 space. If $cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\}) \neq \emptyset \Rightarrow \{x, y\} \subseteq cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\})$. Suppose $\{x, y\} \nsubseteq cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\})$. Let $z \in cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\})$ and $x \notin cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\})$. Then $x \notin cl_{\tilde{\gamma}}(\{y\})$ which implies that $x \in X - cl_{\tilde{\gamma}}(\{y\})$. Let $x \in U, U = X - cl_{\tilde{\gamma}}(\{y\})$ and U is $\tilde{\gamma}$-open. But if $z \in cl_{\tilde{\gamma}}(\{x\})$ then $z \in cl_{\tilde{\gamma}}(\{y\})$ and $z \notin X - cl_{\tilde{\gamma}}(\{y\})$, $z \notin U$. (ie) $cl_{\tilde{\gamma}}(\{x\}) \nsubseteq U$ which is a contradiction to $\tilde{\gamma}$-R_0 space. Conversely let $cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\}) = \emptyset$ or $\{x, y\} \subseteq cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\})$ and let U be a $\tilde{\gamma}$-open such that $x \in U$. Suppose $cl_{\tilde{\gamma}}(\{x\}) \nsubseteq U$ then there exists a element $y \in cl_{\tilde{\gamma}}(\{x\})$ and $y \notin U$ and $cl_{\tilde{\gamma}}(\{y\}) \cap U = \emptyset$. Since $X - U$ is $\tilde{\gamma}$-closed and $y \in (X - U)$. Thus $\{x, y\} \nsubseteq cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\})$ and so $cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\}) \neq \emptyset$, which is a contradiction. Hence X is $\tilde{\gamma}$-R_0.

Theorem 3.4. In a topological space X, the following statements are equivalent.
(i) X is $\tilde{\gamma}$-R_0;
(ii) for each $x \in X, cl_{\tilde{\gamma}}(\{x\}) \subseteq ker_{\tilde{\gamma}}(\{x\})$;
(iii) for each $x, y \in X$ and $y \in ker_{\tilde{\gamma}}(\{x\})$ if and only if $x \in ker_{\tilde{\gamma}}(\{y\})$;

(iv) for each $x, y \in X$ and $y \in cl_{\tilde{\gamma}}(\{x\})$ if and only if $x \in cl_{\tilde{\gamma}}(\{y\})$;
(v) for each $\tilde{\gamma}$-closed set F and a point $x \notin F$, there exists a $U \in \tilde{\gamma}O(X)$ and $F \subseteq U$;
(vi) for each $\tilde{\gamma}$-closed set F can be expressed as $F = \cap\{U : U \in \tilde{\gamma}O(X)$ and $F \subseteq U\}$;
(vii) for each $\tilde{\gamma}$-open set $U, U = \cup\{F : X - F \in \tilde{\gamma}O(X)$ and $F \subseteq U\}$;
(viii) for each $\tilde{\gamma}$-closed set $F, x \notin F$ implies $cl_{\tilde{\gamma}}(\{x\}) \cap F = \emptyset$.

Proof. (i) $\Rightarrow$ (ii). By Definition 3.1, $ker_{\tilde{\gamma}}(\{x\} = \cap\{U : U \in \tilde{\gamma}O(X)$ and $\{x\} \subseteq U\}\}$. Then by (i), each $\tilde{\gamma}$-open set U containing x and contains $cl_{\tilde{\gamma}}(\{x\})$.

(ii) $\Rightarrow$ (iii). For any $x, y \in X$, if $y \in ker_{\tilde{\gamma}}(\{\text{x}\}$, then by Theorem 3.1, $x \in cl_{\tilde{\gamma}}(\{y\})$. By (ii), $x \in ker_{\tilde{\gamma}}(\{\text{y}\})$. Conversely, if $x \in ker_{\tilde{\gamma}}(\{\text{y}\})$, then by Theorem 3.1 ,$y \in cl_{\tilde{\gamma}}(\{x\})$. By (ii) $y \in ker_{\tilde{\gamma}}(\{\text{x}\})$.

(iii) $\Rightarrow$ (iv). For any $x, y \in X$, if $y \in cl_{\tilde{\gamma}}(\{x\})$, by Theorem 3.1, $x \in ker_{\tilde{\gamma}}(\{y\})$. By (iii) $y \in ker_{\tilde{\gamma}}(\{\text{x}\})$. By Theorem 3.1, $x \in cl_{\tilde{\gamma}}(\{y\})$. The converse part is similar.

(iv) $\Rightarrow$ (v). Let F be a $\tilde{\gamma}$-closed set and a point $x \notin F$. Then for any $y \in F$, $cl_{\tilde{\gamma}}(\{x\}) \subseteq F$ and so $x \notin cl_{\tilde{\gamma}}(\{y\})$. By (iv) if $x \notin cl_{\tilde{\gamma}}(\{y\})$ then $y \notin cl_{\tilde{\gamma}}(\{x\})$, implies that there exists a $\tilde{\gamma}$-open set U_y such that $y \in U_y$ and $x \notin U_y$. Let $U = \cup_{y \in F}\{U_y : U_y \in \tilde{\gamma}O(X), y \in U_y$ and $x \notin U_y\ \}$. Then by Theorem 3.4[6], U is $\tilde{\gamma}$-open such that $x \notin U$ and $F \subseteq U$.

(v) $\Rightarrow$ (vi). Let F be $\tilde{\gamma}$-closed set and $H = \cap\{U : U \in \tilde{\gamma}O(X)$ and $F \subseteq U\}$. Clearly $F \subseteq H$. Let $x \in H$. Suppose $x \notin F$. By (v) there exists a $\tilde{\gamma}$-open set U such that $x \notin U$ and $F \subseteq U$, and hence $x \notin H$. Therefore, each $\tilde{\gamma}$-closed set F can be expressed as $F = \cap\{U : U \in \tilde{\gamma}O(X)$ and $F \subseteq U\}$.

(vi) $\Rightarrow$ (vii). It is trivially true as $U = \cup\{F : X - F$ is $\tilde{\gamma}$-open and $F \subseteq U\}$.

(vii) $\Rightarrow$ (viii). Let F be a $\tilde{\gamma}$-closed set and $x \notin F$. Then $X - F = U$, is a $\tilde{\gamma}$-open set containing x. By (vii) we have U can be written as the union of $\tilde{\gamma}$-closed sets and so there is a $\tilde{\gamma}$-closed set H such that $x \in H \subseteq U$ and hence $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$. Thus $cl_{\tilde{\gamma}}(\{x\}) \cap F = \emptyset$.

(viii) $\Rightarrow$ (i). Let U be a $\tilde{\gamma}$-open set and $x \in U$. Then by (viii) there exists a $\tilde{\gamma}$-closed set F such that $x \in F \subseteq U$ and $cl_{\tilde{\gamma}}(\{x\}) \cap F \neq \emptyset$. Therefore $cl_{\tilde{\gamma}}(\{x\}) \subseteq F$ and hence $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$. Thus X is $\tilde{\gamma}$-R_0 space.

Theorem 3.5. For any two points $x, y \in X$ in a $\tilde{\gamma}$-R_0 space we have either $cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\}) = \emptyset$ (or) $cl_{\tilde{\gamma}}(\{x\}) = cl_{\tilde{\gamma}}(\{y\})$.

Proof. Let X be a $\tilde{\gamma}$-R_0 space. Suppose $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$ and $cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\}) \neq \emptyset$. Let $s \in cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\})$ and $x \notin cl_{\tilde{\gamma}}(\{y\})$. Then $x \in X - cl_{\tilde{\gamma}}(\{y\})$, is $\tilde{\gamma}$-open in X. But $cl_{\tilde{\gamma}}(\{x\}) \nsubseteq X - cl_{\tilde{\gamma}}(\{y\})$, since $s \in cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\})$, which is a contradiction to the hypothesis that X is $\tilde{\gamma}$-R_0. Hence we have that either $cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\}) = \emptyset$ (or) $cl_{\tilde{\gamma}}(\{x\}) = cl_{\tilde{\gamma}}(\{y\})$.

Remark 3.1. The converse of the above theorem need not be true, in general.

Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and define an operation $\gamma : \tau \to P(X)$ by

$$A^{\gamma} = \begin{cases} A & \text{if } A = \{b, c\} \\ A \cup \{b, d\} & \text{if } A \neq \{b, c\} \end{cases} \quad \text{for every } A \in \tau.$$

Then $\tilde{\gamma}O(X) = \{\emptyset, X, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and satisfies the condition: for any two points $x, y \in X$, we have either $cl_{\tilde{\gamma}}(\{x\}) \cap cl_{\tilde{\gamma}}(\{y\}) = \emptyset$ (or) $cl_{\tilde{\gamma}}(\{x\}) = cl_{\tilde{\gamma}}(\{y\})$. But X is not $\tilde{\gamma}$-R_0.

Theorem 3.6. For any two points x and y in a topological space X, the following statements are equivalent:
(i) $ker_{\tilde{\gamma}}(\{x\}) \neq ker_{\tilde{\gamma}}(\{y\})$;

(ii) $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$.

Proof. (i) $\Rightarrow$ (ii). Let $ker_{\tilde{\gamma}}(\{x\}) \neq ker_{\tilde{\gamma}}(\{y\})$. Then there exists $z \in ker_{\tilde{\gamma}}(\{x\})$ such that $z \notin ker_{\tilde{\gamma}}(\{y\})$. By Theorem 3.1, $x \in cl_{\tilde{\gamma}}(\{z\})$ and $y \notin cl_{\tilde{\gamma}}(\{z\})$. As $cl_{\tilde{\gamma}}(\{x\}) \subseteq cl_{\tilde{\gamma}}(\{z\})$ we have $y \notin cl_{\tilde{\gamma}}(\{x\})$. Hence $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$.

(ii) $\Rightarrow$ (i). Let $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$. Then there exists $z \in X$ such that $z \in cl_{\tilde{\gamma}}(\{x\})$ and $z \notin cl_{\tilde{\gamma}}(\{y\})$, which implies that there exists a $\tilde{\gamma}$-open set U such that $z \in U$, $y \notin U$ and $x \in U$ implies that $y \notin ker_{\tilde{\gamma}}(\{x\})$. Hence $ker_{\tilde{\gamma}}(\{x\}) \neq ker_{\tilde{\gamma}}(\{y\})$.

Theorem 3.7. Let X be a $\tilde{\gamma}$-R_0 space. Then for any two distinct points $x, y \in X$, $ker_{\tilde{\gamma}}(\{x\}) \neq ker_{\tilde{\gamma}}(\{y\})$ implies $ker_{\tilde{\gamma}}(\{x\}) \cap ker_{\tilde{\gamma}}(\{y\}) = \emptyset$.

Proof. Let X be a $\tilde{\gamma}$-R_0 space and $ker_{\tilde{\gamma}}(\{x\}) \neq ker_{\tilde{\gamma}}(\{y\})$ where $x, y \in X$. Suppose that $ker_{\tilde{\gamma}}(\{x\}) \cap ker_{\tilde{\gamma}}(\{y\}) \neq \emptyset$. Let $s \in ker_{\tilde{\gamma}}(\{x\}) \cap ker_{\tilde{\gamma}}(\{y\})$. Then $s \in ker_{\tilde{\gamma}}(\{x\})$ and $s \in ker_{\tilde{\gamma}}(\{y\})$. By Theorem 3.4.(iii), we have that $x \in ker_{\tilde{\gamma}}(\{s\})$ and $y \in ker_{\tilde{\gamma}}(\{s\})$. Hence $ker_{\tilde{\gamma}}(\{x\}) \subseteq ker_{\tilde{\gamma}}(\{s\}) \subseteq ker_{\tilde{\gamma}}(\{y\})$ and we have $ker_{\tilde{\gamma}}(\{y\}) \subseteq ker_{\tilde{\gamma}}(\{s\}) \subseteq ker_{\tilde{\gamma}}(\{x\})$ implies that $ker_{\tilde{\gamma}}(\{x\}) = ker_{\tilde{\gamma}}(\{y\})$, which is a contradiction. Hence $ker_{\tilde{\gamma}}(\{x\}) \cap ker_{\tilde{\gamma}}(\{y\}) = \emptyset$.

Corollary 3.1. For any pair of points x and y in a topological space X, the following statements are equivalent:
(i) X is $\tilde{\gamma}$-R_0 space;
(ii) for each $\tilde{\gamma}$-closed set $F \subseteq X, F = ker_{\tilde{\gamma}}(F)$;
(iii) for each $\tilde{\gamma}$-closed set $F \subseteq X$ and $x \in F$, $ker_{\tilde{\gamma}}(\{x\}) \subseteq F$;
(iv) for each $x \in X, ker_{\tilde{\gamma}}(\{x\}) \subseteq cl_{\tilde{\gamma}}(\{x\})$.

Proof. (i) $\Rightarrow$ (ii). Let F be a $\tilde{\gamma}$-closed set and $x \notin F$. Then $X - F$ is $\tilde{\gamma}$-open and $x \in X - F$. Since X is $\tilde{\gamma}$-R_0, $cl_{\tilde{\gamma}}(\{x\}) \subseteq X - F$. Therefore $cl_{\tilde{\gamma}}(\{x\}) \cap F = \emptyset$ and by Lemma 3.2, $x \notin ker_{\tilde{\gamma}}(F)$. Hence $ker_{\tilde{\gamma}}(F) \subseteq F$. By Definition 3.1, $F \subseteq ker_{\tilde{\gamma}}(F)$. Thus $F = ker_{\tilde{\gamma}}(F)$.

(ii) $\Rightarrow$ (iii). Let F be a $\tilde{\gamma}$-closed set and $x \in F$. Then {x} $\subseteq F$ and $ker_{\tilde{\gamma}}(\{x\}) \subseteq ker_{\tilde{\gamma}}(F)$. By (ii), we have that $ker_{\tilde{\gamma}}(\{x\}) \subseteq F$.

(iii) $\Rightarrow$ (iv). Since $x \in cl_{\tilde{\gamma}}(\{x\})$ and $cl_{\tilde{\gamma}}(\{x\})$ is a $\tilde{\gamma}$-closed set in X. Then by (iii), $ker_{\tilde{\gamma}}(\{x\}) \subseteq cl_{\tilde{\gamma}}(\{x\})$.

(iv) $\Rightarrow$ (i). Let $x \in cl_{\tilde{\gamma}}(\{y\})$. Then by Theorem 3.1, $y \in ker_{\tilde{\gamma}}(\{x\})$. By (iv) $y \in cl_{\tilde{\gamma}}(\{x\})$. Similarly we can prove if $y \in cl_{\tilde{\gamma}}(\{x\})$ then $x \in ker_{\tilde{\gamma}}(\{y\}$ which implies $x \in cl_{\tilde{\gamma}}(\{y\})$. Then by Theorem 3.4.(iv), X is $\tilde{\gamma}$-R_0 space.

Theorem 3.8. In a topological space X, the following statements are equivalent:
(i) X is $\tilde{\gamma}$-T_1;
(ii) $cl_{\tilde{\gamma}}(\{x\}) = \{x\}$, for all $x \in X$;
(iii) X is $\tilde{\gamma}$-R_0 and $\tilde{\gamma}$-T_0.

Proof. (i) $\Rightarrow$ (ii). Since $\{x\} \subseteq cl_{\tilde{\gamma}}(\{x\})$. If $y \notin \{x\}$, then there exists a $\tilde{\gamma}$-open set U such that $y \in U$, $x \notin U$. Therefore $U \cap \{x\} = \emptyset$ and hence $y \notin cl_{\tilde{\gamma}}(\{x\})$.

(ii) $\Rightarrow$ (iii). Let $x, y \in X$ with $x \neq y$. Then $\{x\}$ and $\{y\}$ are $\tilde{\gamma}$-closed sets and hence $X - \{x\}$ is $\tilde{\gamma}$-open set containing y but not x which implies X is $\tilde{\gamma}$-T_0. Suppose that U is $\tilde{\gamma}$-open set and $x \in U$. Then by (ii), $cl_{\tilde{\gamma}}(\{x\}) = \{x\} \subseteq U$. Hence X is $\tilde{\gamma}$-R_0.

(iii) $\Rightarrow$ (i). Let $x, y \in X$ with $x \neq y$. Then there exits $\tilde{\gamma}$-open set U such that $x \in U$ and $y \notin U$ (say) which implies that $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$ and so $y \notin cl_{\tilde{\gamma}}(\{x\})$. Hence $x \in U$, U is $\tilde{\gamma}$-open, $y \notin U$ and $y \in X - cl_{\tilde{\gamma}}(\{x\})$, which is $\tilde{\gamma}$-open, $x \notin X - cl_{\tilde{\gamma}}(\{y\})$. Hence X is $\tilde{\gamma}$-T_1.

Definition 3.2. A topological space X is said to be $\tilde{\gamma}$-R_1 if for each $x, y \in X, cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$, there exists $\tilde{\gamma}$-open sets U, V such that $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$ and $cl_{\tilde{\gamma}}(\{y\}) \subseteq V$ and $U \cap V = \emptyset$.

Example 3.2. Let $X = \{a, b, c, d\}$, $\tau = P(X)$ and define an operation $\gamma : \tau \to P(X)$ by

$$A^{\gamma} = \begin{cases} A \cup \{c, d\} & \text{if } A = \{a\}(or)\{b\} \\ A \cup \{a, b\} & \text{if } A = \{c\}(or)\{d\} \\ A & \text{Otherwise} \end{cases} \quad \text{for every } A \in \tau.$$

Then $\tilde{\gamma}O(X) = \{\emptyset, X, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Hence X is $\tilde{\gamma}$-R_1.

Theorem 3.9. If X is $\tilde{\gamma}$-R_1, then it is $\tilde{\gamma}$-R_0.

Proof. Let U be a $\tilde{\gamma}$-open set and $x \in U$. If $y \notin U$, since $x \notin cl_{\tilde{\gamma}}(\{y\})$, we have that $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$. So there exists a $\tilde{\gamma}$-open set V such that $cl_{\tilde{\gamma}}(\{y\}) \subseteq V$ and $x \notin V$, which implies that $y \notin cl_{\tilde{\gamma}}(\{x\})$. Hence $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$. Hence X is $\tilde{\gamma}$-R_0.

Remark 3.2. The converse of the above Theorem 3.9 need not be true in general.

Let $X = \{a, b, c, d\}$, $\tau = P(X)$ and define an operation $\gamma : \tau \to P(X)$ by

$$A^{\gamma} = \begin{cases} A & \text{if } A = \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\} \\ X & \text{Otherwise} \end{cases} \quad \text{for every } A \in \tau.$$

Then $\tilde{\gamma}O(X) = \{\emptyset, X, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Hence X is $\tilde{\gamma}$-R_0 but not $\tilde{\gamma}$-R_1.

Theorem 3.10. In a topological space X, the following statements are equivalent:
(i) X is $\tilde{\gamma}$-T_2;
(ii) X is $\tilde{\gamma}$-R_1 and $\tilde{\gamma}$-T_1;
(iii) X is $\tilde{\gamma}$-R_1 and $\tilde{\gamma}$-T_0.

Proof. (i) $\Rightarrow$ (ii). Let X be a $\tilde{\gamma}$-T_2 space. Then X is clearly $\tilde{\gamma}$-T_1. Now if $x, y \in X$ with $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$ then there exists $\tilde{\gamma}$-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. Hence by Theorem 3.8 $cl_{\tilde{\gamma}}(\{x\}) = \{x\} \subseteq U$ and $cl_{\tilde{\gamma}}(\{y\}) = \{y\} \subseteq V$ and $U \cap V = \emptyset$. Then X is $\tilde{\gamma}$-R_1.

(ii) $\Rightarrow$ (iii). It is trivially true.

(iii) $\Rightarrow$ (i). Let X be $\tilde{\gamma}$-R_1 and $\tilde{\gamma}$-T_0. By Theorem 3.9, X is $\tilde{\gamma}$-$R_1 \Rightarrow X$ is $\tilde{\gamma}$-R_0. By Theorem 3.8, X is $\tilde{\gamma}$-R_0 and $\tilde{\gamma}$-$T_0 \Rightarrow X$ is $\tilde{\gamma}$-T_1. Let $x, y \in X$ with $x \neq y$. Then $cl_{\tilde{\gamma}}(\{x\}) = \{x\} \neq \{y\} = cl_{\tilde{\gamma}}(\{y\})$. As X is $\tilde{\gamma}$-R_1, there exists $\tilde{\gamma}$-open sets U, V such that $cl_{\tilde{\gamma}}(\{x\}) = \{x\} \subseteq U$, $cl_{\tilde{\gamma}}(\{y\}) = \{y\} \subseteq V$ and $U \cap V = \emptyset$. Thus X is $\tilde{\gamma}$-T_2.

Theorem 3.11. In a topological space X, the following statements are equivalent:
(i) X is $\tilde{\gamma}$-R_1;
(ii) for any $x, y \in X$ one of the following holds:
  (a) for $\tilde{\gamma}$-open set $U, x \in U$ if and only if $y \in U$;
  (b) there exists $\tilde{\gamma}$-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.
(iii) if $x, y \in X$ such that $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$, then there exists $\tilde{\gamma}$-closed sets F_1 and F_2 such that $x \in F_1$, $y \notin F_1, y \in F_2, x \notin F_2$ and $X = F_1 \cup F_2$.

Proof. (i) $\Rightarrow$ (ii). Let $x, y \in X$. Then $cl_{\tilde{\gamma}}(\{x\}) = cl_{\tilde{\gamma}}(\{y\})$ (or) $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$. Suppose $cl_{\tilde{\gamma}}(\{x\}) = cl_{\tilde{\gamma}}(\{y\})$ and U, $\tilde{\gamma}$-open set. Then $x \in U$ implies that $y \in cl_{\tilde{\gamma}}(\{y\}) = cl_{\tilde{\gamma}}(\{x\}) \subseteq U$. Hence $y \in U$. Similarly, we can prove if $y \in U$ then $x \in U$. Suppose $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$. Then there exist $\tilde{\gamma}$-open sets U, V such that $x \in cl_{\tilde{\gamma}}(\{x\}) \subseteq U$ and $y \in cl_{\tilde{\gamma}}(\{y\}) \subseteq V$ and $U \cap V = \emptyset$.

(ii) $\Rightarrow$ (iii). Let $x, y \in X$ such that $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$. Then $x \notin cl_{\tilde{\gamma}}(\{y\})$, so that there exist a $\tilde{\gamma}$-open set G such that $x \in G$ and $y \notin G$. Thus by (ii), there exists $\tilde{\gamma}$-open sets U and V such that $x \in U$ and $y \in V$ and $U \cap V = \emptyset$. Put $F_1 = X - V$ and $F_2 = X - U$. Then F_1 and F_2 are $\tilde{\gamma}$-closed sets and $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$.

(iii) $\Rightarrow$ (i). Let U be $\tilde{\gamma}$-open set and $x \in U$. Then $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$. In fact, otherwise there exists $y \in cl_{\tilde{\gamma}}(\{x\}) \cap (X - U)$. Then $cl_{\tilde{\gamma}}(\{x\}) \neq cl_{\tilde{\gamma}}(\{y\})$ and so by (iii), there exists F_1 and F_2 which are $\tilde{\gamma}$-closed sets such that $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$. Then $y \in F_2 - F_1 = X - F_1$ and $x \notin X - F_1$, where $X - F_1, \tilde{\gamma}$-open set which is a contradiction to the fact that $y \in cl_{\tilde{\gamma}}(\{x\})$. Hence $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$. Thus X is $\tilde{\gamma}$-R_0. To show X is $\tilde{\gamma}$-R_1 assume that $a, b \in X$ with $cl_{\tilde{\gamma}}(\{a\}) \neq cl_{\tilde{\gamma}}(\{b\})$. Then there exists $\tilde{\gamma}$-closed sets P_1 and P_2 such that $x \in P_1$, $y \notin P_1$, $y \in P_2$, $x \notin P_2$ and $X = P_1 \cup P_2$. Thus $a \in P_1 - P_2 \in \tilde{\gamma}O(X)$, $b \in P_2 - P_1 \in \tilde{\gamma}O(X)$. So $cl_{\tilde{\gamma}}(\{a\}) \subseteq P_1 - P_2$ and $cl_{\tilde{\gamma}}(\{b\}) \subseteq P_2 - P_1$. Thus X is $\tilde{\gamma}$-R_1.

Theorem 3.12. (i) A topological space X is $\tilde{\gamma}$-T_2 if and only if for $x, y \in X$ with $x \neq y$ there exists $\tilde{\gamma}$-closed sets F_1 and F_2 such that $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$.
(ii) A topological space X is $\tilde{\gamma}$-R_1 if and only if $x, y \in X$, with $ker_{\tilde{\gamma}}(\{x\}) \neq ker_{\tilde{\gamma}}(\{y\}$, there exists $\tilde{\gamma}$-open sets U, V such that $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$ and $cl_{\tilde{\gamma}}(\{y\}) \subseteq V$ and $U \cap V = \emptyset$.

Proof. (i) Follows from Theorems 3.10 and 3.11.
(ii) Follows from Theorem 3.6 and Definition 3.2.

Definition 3.3 Let X be a topological space. Then a net $\{x_\alpha\}_{\alpha \in J}$ in X is said to $\tilde{\gamma}$-converge to a point x in X if the net is eventually in every $\tilde{\gamma}$-open set containing x.

Lemma 3.1 Let x, y be two points in a topological space X. If every net in X which $\tilde{\gamma}$-converges to y also $\tilde{\gamma}$-converges to x, then $x \in cl_{\tilde{\gamma}}(\{y\})$.

Proof. Let us consider the net $x_n = y$ for each n $\in N(N$ - natural numbers). Clearly the net $\tilde{\gamma}$-converges to y and hence $\tilde{\gamma}$-converges to x. Thus if U is $\tilde{\gamma}$-open set with $x \in U$, then $\{x_n\}_{n \in N}$ is eventually in $U \Rightarrow y \in U$ Thus $x \in cl_{\tilde{\gamma}}(\{y\})$.

Theorem 3.13. Let X be a topological space. Then X is $\tilde{\gamma}$-R_0 if and only if for every $x, y \in X$, $y \in cl_{\tilde{\gamma}}(\{y\}) \Leftrightarrow$ every net in X is $\tilde{\gamma}$-converging to y also $\tilde{\gamma}$-converges to x.

Proof. Let X be $\tilde{\gamma}$-R_0. Suppose $y \in cl_{\tilde{\gamma}}(\{x\})$. To prove every net in X is $\tilde{\gamma}$-converging to y also $\tilde{\gamma}$-converges to x. $y \in cl_{\tilde{\gamma}}(\{y\})$ for some $x, y \in X$ and let $\{x_\alpha\}_{\alpha \in J}$ be a net in X is $\tilde{\gamma}$-converging to y. Since $y \in cl_{\tilde{\gamma}}(\{x\})$, $cl_{\tilde{\gamma}}(\{x\}) = cl_{\tilde{\gamma}}(\{y\})$. Let U be $\tilde{\gamma}$-open set such that $x \in U$. Then $y \in U$ and hence there exists $\alpha_0 \in J$ such that if $\alpha \geq \alpha_0$ then $x_\alpha \in U$. Thus $\{x_\alpha\}_{\alpha \in J}$ $\tilde{\gamma}$-converges to x. On the other hand, suppose that every net in X is $\tilde{\gamma}$-converging to y, $\tilde{\gamma}$-converges to x. By lemma 3.1, $x \in cl_{\tilde{\gamma}}(\{y\})$. By Theorem 3.5. $cl_{\tilde{\gamma}}(\{x\}) = cl_{\tilde{\gamma}}(\{y\})$ and hence $y \in cl_{\tilde{\gamma}}(\{x\})$. Conversely, to prove X to be $\tilde{\gamma}$-R_0, let U be $\tilde{\gamma}$-open set and $x \in U$. Let $y \in X - U$. For each $n \in N$, let $x_n = y$. Then the net $\{x_n\}_{n \in N}$ $\tilde{\gamma}$-converges to y, but $\{x_n\}$ is not $\tilde{\gamma}$-convergent to x. Thus $y \notin cl_{\tilde{\gamma}}(\{x\})$. Hence $cl_{\tilde{\gamma}}(\{x\}) \subseteq U$.

4 $(\gamma, \tilde{\gamma})$-normal space and $(\gamma, \tilde{\gamma})$-regular space

Definition 4.1. A topological space X is said to be $(\gamma, \tilde{\gamma})$-normal if for any pair of disjoint γ-closed sets A, B of X, there exists disjoint $\tilde{\gamma}$-open sets U, V of X such that $A \subseteq U$ and $B \subseteq V$.

Example 4.1. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and define an operation $\gamma : \tau \to P(X)$ by

$$A^\gamma = \begin{cases} A & \text{if } A = \{b, c\}(or)\{a, b, c\} \\ cl(A) & \text{Otherwise} \end{cases} \quad \text{for every } A \in \tau.$$

Then $\tilde{\gamma}O(X) = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. Hence X is $(\gamma, \tilde{\gamma})$-normal.

Theorem 4.1. Let X be a topological space. Then the following properties are equivalent:
(i) X is $(\gamma, \tilde{\gamma})$-normal;
(ii) for each γ-closed set A and for each γ-open set V of A, there exists a $\tilde{\gamma}$-open set U of A such that $cl_{\tilde{\gamma}}(U) \subseteq V$;
(iii) for each pair of disjoint γ-closed sets A and B in X, there exists a $\tilde{\gamma}$-open set U of A such that $cl_{\tilde{\gamma}}(U) \cap B = \emptyset$;
(iv) for any pair of disjoint γ-closed sets A, B of X, there exists disjoint g$\tilde{\gamma}$-open sets U, V such that $A \subseteq U$ and $B \subseteq V$;
(v) for any γ-closed set A and any γ-open set V containing A, there exists g$\tilde{\gamma}$-open set U such that $A \subseteq U \subseteq cl_{\tilde{\gamma}}(U) \subseteq V$.

Proof. (i) $\Rightarrow$ (ii). Let X be a $(\gamma, \tilde{\gamma})$-normal space and A be any γ-closed set and V be any γ-open set of A. Now A and $X - V$ are γ-closed in X and $A \subseteq V$ implies that $A \cap (X - V) = \emptyset$. Since X is $(\gamma, \tilde{\gamma})$-normal, there exists $\tilde{\gamma}$-open set U of A and $\tilde{\gamma}$-open set W of $X - V$ such that $U \cap W = \emptyset \Rightarrow U \subseteq X - W \Rightarrow cl_{\tilde{\gamma}}(U) \subseteq cl_{\tilde{\gamma}}(X - W) = X - W$ (since $X - W$ is $\tilde{\gamma}$-closed) $\Rightarrow cl_{\tilde{\gamma}}(U) \cap W = \emptyset$. But $cl_{\tilde{\gamma}}(U) \cap (X - V) \subseteq cl_{\tilde{\gamma}}(U) \cap W = \emptyset \Rightarrow cl_{\tilde{\gamma}}(U) \cap (X - V) = \emptyset \Rightarrow cl_{\tilde{\gamma}}(U) \subseteq V$.

(ii) $\Rightarrow$ (iii). Let A, B be disjoint γ-closed sets in X. Since $A \cap B = \emptyset$ we have $A \subseteq X - B$, where $X - B$ is γ-open. Hence $X - B$ is a γ-open set containing the γ-closed set A. By (ii) there exists $\tilde{\gamma}$-open set U of A such that $cl_{\tilde{\gamma}}(U) \subseteq X - B$. Hence $cl_{\tilde{\gamma}}(U) \cap B = \emptyset$.

(iii) $\Rightarrow$ (i). Let A, B be disjoint γ-closed sets in X. By (iii), there exists a $\tilde{\gamma}$-open set U_1 of A such that $cl_{\tilde{\gamma}}(U_1) \cap B = \emptyset \Rightarrow B \subseteq X - cl_{\tilde{\gamma}}(U_1)$. Take $U_2 = X - cl_{\tilde{\gamma}}(U_1)$, then U_2 is a $\tilde{\gamma}$-open set containing the γ-closed set B. Also $U_1 \cap U_2 = U_1 \cap (X - cl_{\tilde{\gamma}}(U_1)) = \emptyset$. Hence X is $(\gamma, \tilde{\gamma})$-normal.

(i) $\Rightarrow$ (iv). Follows from the definition of $(\gamma, \tilde{\gamma})$-normal and Remark 2.1.

(iv) $\Rightarrow$ (v). Let A be any γ-closed set and V a γ-open set containing A. Since A and $X - V$ are disjoint γ-closed sets of X, there exists $g\tilde{\gamma}$-open sets U and W of X such that $A \subseteq U$, $X - V \subseteq W$ and $U \cap W = \emptyset$. Therefore by definition of $g\tilde{\gamma}$-open, we have that $X - V \subseteq int_{\tilde{\gamma}}(W)$. Since $U \cap int_{\tilde{\gamma}}(W) = \emptyset$, we have that $cl_{\tilde{\gamma}}(U) \cap int_{\tilde{\gamma}}(W) = \emptyset$ and hence $cl_{\tilde{\gamma}}(U) = X - int_{\tilde{\gamma}}(W) \subseteq V$. Therefore $A \subseteq U \subseteq cl_{\tilde{\gamma}}(A) \subseteq V$.

(v) $\Rightarrow$ (i). Let A and B be any disjoint γ-closed sets of X. Since $X - B$ is a γ-open set containing A and by (v), there exists a $g\tilde{\gamma}$-open set G such that $A \subseteq G \subseteq cl_{\tilde{\gamma}}(G) \subseteq X - B$. By the definition of $g\tilde{\gamma}$-open, we have that $A \subseteq int_{\tilde{\gamma}}(G)$. Put $U = int_{\tilde{\gamma}}(G)$ and $V = X - cl_{\tilde{\gamma}}(G)$. This implies that U and V are disjoint $\tilde{\gamma}$-open sets such that $A \subseteq U$ and $B \subseteq V$. Therefore X is $(\gamma, \tilde{\gamma})$-normal.

Theorem 4.2. Let $f : X \to Y$ be a mapping. If f is (γ, β)-continuous, $(\tilde{\gamma}, \tilde{\beta})$-open, surjective and X is $(\gamma, \tilde{\gamma})$-normal, then Y is $(\beta, \tilde{\beta})$-normal.

Proof. Let A and B be any two disjoint β-closed sets in Y. Since f is (γ, β)-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint γ-closed in X. As X is $(\gamma, \tilde{\gamma})$-normal, there exist disjoint $\tilde{\gamma}$-open sets U and V of X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$ and $U \cap V = \emptyset$. Since f is $(\tilde{\gamma}, \tilde{\beta})$-open and surjective we have that $f(U)$ and $f(V)$ are $\tilde{\beta}$-open sets in Y such that $A \subseteq f(U)$ and $B \subseteq f(V)$ and $f(U) \cap f(V) = \emptyset$. Hence Y is $(\beta, \tilde{\beta})$-normal.

Theorem 4.3. Let $f : X \to Y$ be a mapping. If f is (γ, β)-closed and $(\tilde{\gamma}, \tilde{\beta})$-continuous, injective and Y is $(\beta, \tilde{\beta})$-normal, then X is $(\gamma, \tilde{\gamma})$-normal.

Proof. Let A and B be any two disjoint γ-closed in X. Since f is (γ, β)-closed, $f(A)$ and $f(B)$ are disjoint β-closed sets in Y. As Y is $(\beta, \tilde{\beta})$-normal, there exist disjoint $\tilde{\beta}$-open sets U and V of Y such that $f(A) \subseteq U$ and $f(B) \subseteq V$ and $U \cap V = \emptyset$. Since f is $(\tilde{\gamma}, \tilde{\beta})$-continuous and injective we have that $f^{-1}(U)$, $f^{-1}(V)$ are $\tilde{\gamma}$-open sets in X and $A \subseteq f^{-1}(U)$, $B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence X is $(\gamma, \tilde{\gamma})$-normal.

Definition 4.2. A topological space X is said to be $(\gamma, \tilde{\gamma})$-regular if for each γ-closed set F of X and each point $x \in X - F$, there exist disjoint $\tilde{\gamma}$-open sets U, V such that $F \subseteq U$ and $x \in V$.

Example 4.2. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$ and define an operation $\gamma : \tau \to P(X)$ by

$$A^\gamma = \begin{cases} A \cup \{c\} & \text{if } A = \{a, b, d\} \\ A & \text{if } A \neq \{a, b, d\} \end{cases} \quad \text{for every } A \in \tau.$$

Then $\tilde{\gamma}O(X) = \{\emptyset,\ X,\ \{a\},\ \{b\},\ \{c\},\ \{a, b\},\ \{a, c\},\ \{a, d\},\ \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Hence X is $(\gamma, \tilde{\gamma})$-regular.

Theorem 4.4. Let X be a topological space. Then the following properties are equivalent:
(i) X is $(\gamma, \tilde{\gamma})$-regular;
(ii) for each $x \in X$ and each γ-open set U of x, there exists a $\tilde{\gamma}$-open set V of x such that $cl_{\tilde{\gamma}}(V) \subseteq U$;
(iii) for each γ-closed set F of X, $\cap\{cl_{\tilde{\gamma}}(V) : F \subseteq V, V \in \tilde{\gamma}O(X)\} = F$;
(iv) for each $A \subseteq X$ and each γ-open set U with $A \cap U \neq \emptyset$, there exists a $\tilde{\gamma}$-open set V such that $A \cap V \neq \emptyset$ and $cl_{\tilde{\gamma}}(V) \subseteq U$;
(v) for each $A \subseteq X$ and each γ-closed subset F of X with $A \cap F = \emptyset$, there exist V, $W \in \tilde{\gamma}O(X)$ such that $A \cap V \neq \emptyset$, $F \subseteq W$ and $W \cap V = \emptyset$;
(vi) for each γ-closed set F and $x \notin F$, there exists a $\tilde{\gamma}$-open set G and a $g\tilde{\gamma}$-open set V such that $x \in G$, $F \subseteq V$ and $G \cap V = \emptyset$;
(vii) for each $A \subseteq X$ and each γ-closed set F with $A \cap F = \emptyset$, there exists a $\tilde{\gamma}$-open set G and a $g\tilde{\gamma}$-open set V such that $A \cap G \neq \emptyset$, $F \subseteq V$ and $G \cap V = \emptyset$;
(viii) for each γ-closed set F of X, $F = \cap\{cl_{\tilde{\gamma}}(V) : F \subseteq V, V \in G\tilde{\gamma}O(X)\}$.

Proof. (i) $\Rightarrow$ (ii). Let $x \notin X - U$ and U be a γ-open set containing x. Then by (i), there exists G, $V \in \tilde{\gamma}O(X)$ such that $X - U \subseteq G$, $x \in V$ and $G \cap V = \emptyset$. Therefore $V \subseteq X - G$ and $x \in V \subseteq cl_{\tilde{\gamma}}(V) \subseteq X - G \subseteq U$.

(ii) $\Rightarrow$ (iii). Let $X - F$ be a γ-open set containing x. Then by (ii), there exists a $\tilde{\gamma}$-open set G of x such that $x \in G \subseteq cl_{\tilde{\gamma}}(G) \subseteq X - F$. This implies that $F \subseteq X - cl_{\tilde{\gamma}}(G) = V$, $V \in \tilde{\gamma}O(X)$ and $V \cap G = \emptyset$. Then by Remark 2.1, $x \notin cl_{\tilde{\gamma}}(V)$ and hence $F \supseteq \{cl_{\tilde{\gamma}}(V) : F \subseteq V,\ V \in \tilde{\gamma}O(X)\}$.

(iii) $\Rightarrow$ (iv). Let U be a γ-open set with $x \in U \cap A$. Then $x \notin X - U$ and by (iii), there exists a $\tilde{\gamma}$-open set W such that $X - U \subseteq W$ and $x \notin cl_{\tilde{\gamma}}(W)$. We put $V = X - cl_{\tilde{\gamma}}(W)$, which is a $\tilde{\gamma}$-open set containing x and hence $V \cap A \neq \emptyset$. Now $V \subseteq X - W$ and so that $cl_{\tilde{\gamma}}(V) \subseteq X - W \subseteq U$.

(iv) $\Rightarrow$ (v). Let $A \subseteq X$ and F be a γ-closed set in X with $A \cap F = \emptyset$. Then $X - F$ is γ-open and $(X - F) \cap A \neq \emptyset$. Then by (iv), there exists $V \in \tilde{\gamma}O(X)$ such that $A \cap V \neq \emptyset$ and $cl_{\tilde{\gamma}}(V) \subseteq X - F$. If we put $W = X - cl_{\tilde{\gamma}}(V)$, then $W \in \tilde{\gamma}O(X)$, $F \subseteq W$ and $W \cap V = \emptyset$.

(v) $\Rightarrow$ (i). Let F be a γ-closed set not containing x. Then by (v), there exist V, $W \in \tilde{\gamma}O(X)$ such that $F \subseteq W$ and $x \in V$ and $W \cap V = \emptyset$.

(i) $\Rightarrow$ (vi). Follows from the definition of $(\gamma, \tilde{\gamma})$-regular and by Remark 2.2.

(vi) $\Rightarrow$ (vii). Let $A \subseteq X$ and F be a γ-closed set in X with $A \cap F = \emptyset$. For $a \in A$, $a \notin X - A \Rightarrow a \notin F$ and hence by (vi), there exists $G \in \tilde{\gamma}O(X)$ and a $g\tilde{\gamma}$-open set V such that $a \in G$, $F \subseteq V$ and $G \cap V = \emptyset$. Hence $A \cap G \neq \emptyset$.

(vii) $\Rightarrow$ (i). Let $x \notin F$, where F is γ-closed. Since $\{x\} \cap F = \emptyset$, by (vii), there exists $G \in \tilde{\gamma}O(X)$ and a $g\tilde{\gamma}$-open set W such that $x \in G$, $F \subseteq W$ and $G \cap W = \emptyset$. Now put $V = int_{\tilde{\gamma}}(W)$. By definition of $g\tilde{\gamma}$-open sets, we get $F \subseteq V$ and $V \cap G = \emptyset$.

(iii) $\Rightarrow$ (viii). We have that $F \subseteq \cap\{cl_{\tilde{\gamma}}(V) : F \subseteq V, V \in G\tilde{\gamma}O(X)\} \subseteq \cap\{cl_{\tilde{\gamma}}(V) : F \subseteq V, V \in \tilde{\gamma}O(X)\} = F$.

(viii) $\Rightarrow$ (i). Let F be a γ-closed set in X not containing x. Then by (viii), there exists a $g\tilde{\gamma}$-open set V such that $F \subseteq V$ and $x \in X - int_{\tilde{\gamma}}(V)$. Since F is γ-closed and V is $g\tilde{\gamma}$-open, $F \subseteq int_{\tilde{\gamma}}(V)$. Take $W = int_{\tilde{\gamma}}(V)$. Then $F \subseteq W$, $x \in G = X - cl_{\tilde{\gamma}}(W)$ and $G \cap W = \emptyset$.

Theorem 4.5. $f : X \to Y$ be a mapping. If f is (γ, β)-continuous, $(\tilde{\gamma}, \tilde{\beta})$-open, surjective and X is $(\gamma, \tilde{\gamma})$-regular, then Y is $(\beta, \tilde{\beta})$-regular.

Proof. Let $y \in Y$ and F be any β-closed in Y with $y \notin F$. Since f is (γ, β)-continuous, $f^{-1}(F)$ is γ-closed in X. Since f is surjective, let $f(x) = y$, then $x = f^{-1}(y) \Rightarrow x \notin f^{-1}(F)$. Since X is $(\gamma, \tilde{\gamma})$-regular there exists $\tilde{\gamma}$-open sets U and V in X such that $x \in U$ and $f^{-1}(F) \subseteq V$ and $U \cap V = \emptyset$. As f is $(\tilde{\gamma}, \tilde{\beta})$-open, $f(U)$ and $f(V)$ are $\tilde{\beta}$-open in Y. Since f is surjective, $f(U) \cap f(V) = f(U \cap V) = \emptyset$. Hence Y is $(\beta, \tilde{\beta})$-regular.

Theorem 4.6. Let $f : X \to Y$ be a mapping. If f is (γ, β)-closed and $(\tilde{\gamma}, \tilde{\beta})$-continuous, injective and Y is $(\beta, \tilde{\beta})$-regular, then X is $(\gamma, \tilde{\gamma})$-regular.

Proof. Let F be any γ-closed in X with $x \in X$ and $x \notin F$. Since f is (γ, β)-closed, $f(F)$ is β-closed in Y, $f(x) \in Y$ and $f(x) \notin f(F)$. Since Y is $(\beta, \tilde{\beta})$-regular there exists $\tilde{\beta}$-open sets U and V in Y such that $f(x) \in U$ and $f(F) \subseteq V$ and $U \cap V = \emptyset \Rightarrow x \in f^{-1}(U)$ and $F \subseteq f^{-1}(V)$. As f is $(\tilde{\gamma}, \tilde{\beta})$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $\tilde{\gamma}$-open in X. Since f is injective, $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = \emptyset$. Hence X is $(\gamma, \tilde{\gamma})$-regular.

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